On the Capillary Phenomena of Jets

Lord Rayleigh (1879)

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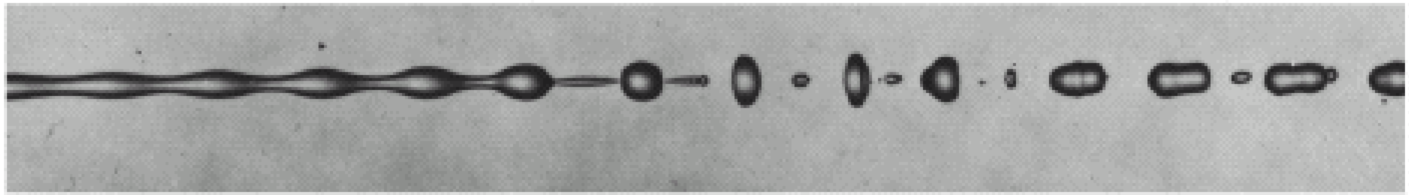
HML Reading Group: Summer 2007

The Problem



- Perturbations on jet grow downstream and lead to break up of the jet into droplets.
- Satellite droplets are formed due to the localized pinching of the jet at the ligaments that connect two droplets just prior to break-up.

Bidone (1823): Shape of the jet produced from orifices of different forms.

Savart (1833): Noticed decay into drops. Studied the laws governing it. Imposed periodic oscillations to obtain disturbances on the surface of the jet with the same frequency.

Plateau (1843): Perturbations of long wavelength reduce surface area and are favored by surface tension

Rayleigh (1879): Realized surface tension had to work against inertia in order to bring about jet break up. Introduced photographic techniques. Viscous jets.

Chandrashekhar (1961): Dispersion relation considering the Navier-Stokes case.

Flow Regimes

$$Oh = \frac{Viscous\ time}{inertial\ time} = \frac{\mu R/\gamma}{\left(\rho R^3/\gamma\right)}$$

$Oh > 1$ $\qquad$ $Oh < 1$

- Viscous thread regime
  - Balance between surface tension and viscosity

- Inviscid regime
  - Balance between inertia and surface tension.

High viscosity $\qquad$ Low viscosity

$$\omega = \omega_0 \left\{ \left[\frac{1}{2} x^2 \left(1 - x^2\right) + \frac{1}{4} Oh^{-2} x^4 \right]^{1/2} - \frac{3}{2} Oh^{-1} x^2 \right\}$$

High viscosity:

$$\omega = \omega_\nu \frac{1}{6}\left(1 - x^2\right)$$

$$\omega_\nu = \gamma/\mu R$$

Good agreement for all Reynolds numbers,

7% off when $\mu \rightarrow 0$

$$\mathrm{Re} = \frac{u_0 R}{(\mu/\rho)} \qquad u_0 = \sqrt{\gamma/\rho R} \qquad x = 2\pi R/\lambda \qquad \omega_0 = \sqrt{\left(\gamma/\rho R^3\right)}$$

Eggers-Weber

Low viscosity:

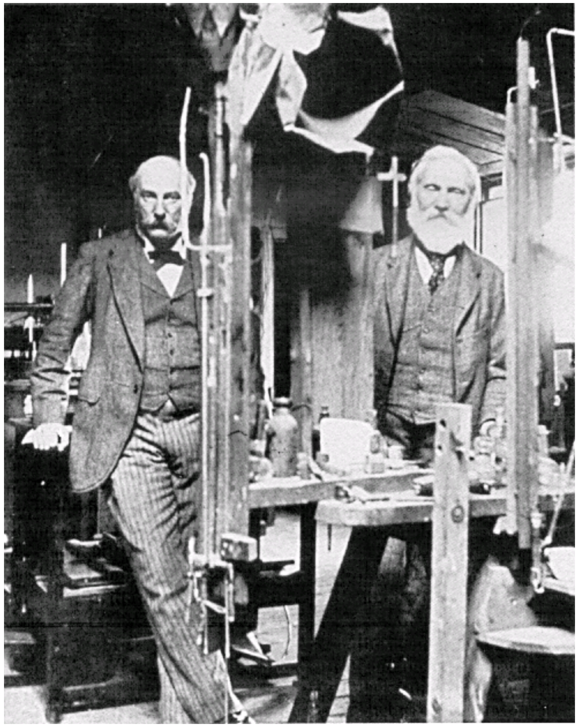
$$x^2 = \frac{1}{2 + \sqrt{18/\mathrm{Re}}}$$

$\mathrm{Re} \rightarrow \infty$

$x \rightarrow 0.707$

$x_{\mathrm{R}} = 0.68$

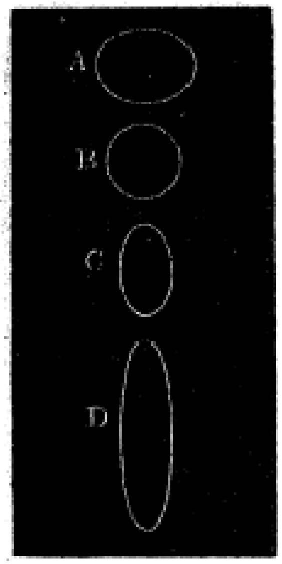
John William Strutt, Lord Rayleigh

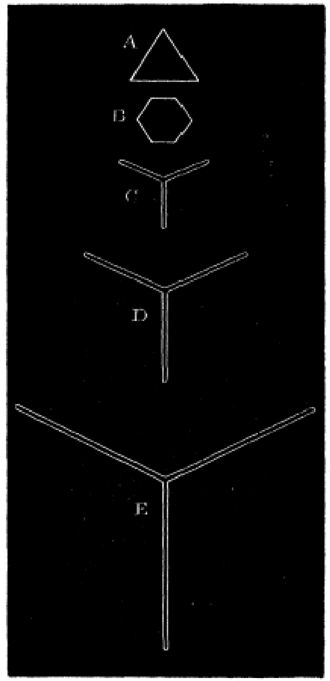


Lord Rayleigh and Kelvin

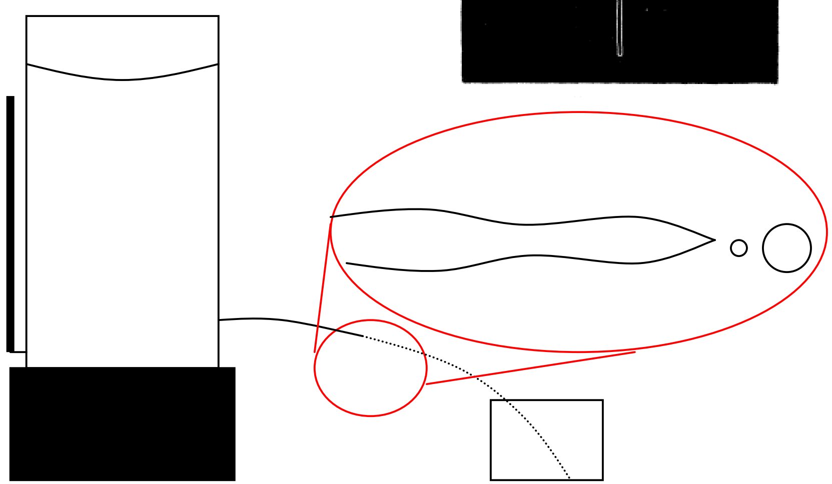
- John William Strutt, born as 3rd Baron Rayleigh→ Suffered frailty and poor health at early ages→ Studied mathematics at Trinity College, Cambridge.
- Succeeded J.C. Maxwell as 2nd Cavendish Professor of Physics, at Cambridge→ Started the discipline of experimental physics at Cambridge.
- Most noted for his book "Theory of Sound", although his research interest was very diverse.
- Read Helmholtz's works very thoroughly at early stages→ used concepts in in his first elaborate research report in Phil.Tran. Royl.Soc, in 1830→ introduced the concept of acoustical conductivity which has remained a standard acoustical quantity ever since.
- Won the **Nobel Prize** for the discovery of Argon (along with William Ramsay) in 1904.
- Notable students: J.J. Thomson (electron, isotopes, mass spectrometer: Nobel Prize 1906), G. P. Thomson (wave properties of electron: Nobel Prize 1937), J. C. Bose (pioneer in radio and microwave optics)

Rayleigh's problem



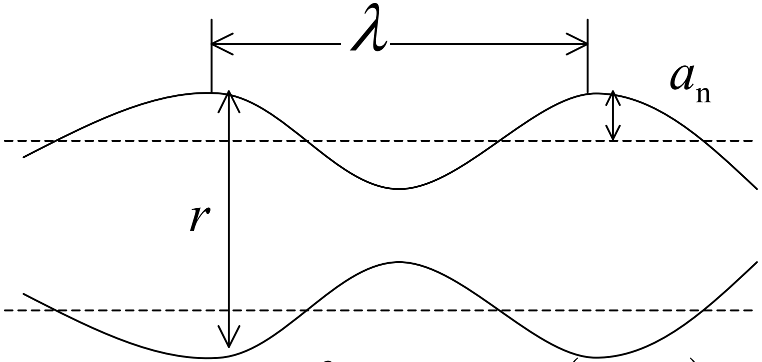


Admitting the substantial accuracy of Bidone's explanation of the formation and primary expansion of the sheets or excrescences, we have to inquire into the cause of the subsequent contraction. Bidone attributes it to the viscosity of the fluid, which may certainly be put out of the question. In Magnus's view the cause is "cohesion;" but he does not explain what is to be understood under this designation, and it is doubtful whether he had a clear idea upon the subject. The true explanation appears to have been first given by Buff,† who refers the phenomenon distinctly to the capillary force. Under the operation of this force the fluid behaves as if enclosed in an envelope of constant tension, and the recurrent form of the jet is due to vibrations of the fluid column about the circular figure of equilibrium, superposed upon the general progressive motion.



- Conducts experiments with different sections, elliptical, triangular, square, circular
- Arrives at the law that the wave-length scales as the square-root of the pressure head.
- Deviations at high pressure attributed to deviation from the assumption of "isochronism" which is expected strictly at small amplitude vibrations.

Linear stability on inviscid jets



$r = a_0 + a_n \cos n\theta \qquad a_n \propto \cos(pt - \varepsilon)$

a_0 average radius of the jet.

$$p = 2\pi\omega = (\pi/A)^{3/4} (T/\rho)^{1/2} \sqrt{(n-1)n(n+1)}$$

*"The case of **n=1** corresponds to a displacement of the jet as a whole without any change to the form of the boundary. There is **no potential energy** and the frequency of vibration is zero."*

$$P = \frac{1}{4}\pi a^{-1} T \left(k^2 a^2 + n^2 - 1\right)\alpha_n^2$$

"When n=1, there is no force of restitution for a case of a displacement in two dimension."

$$\lambda = A^{1/2} f\left(T/PA^{1/2}\right)$$

For the specific case of water jets.

$$\lambda = \frac{\sqrt{(2gh)}}{3.38\sqrt{(n-1)n(n+1)}} A^{3/4}$$

Nearly agrees with experimental observations.

"The discrepancies are to be attributed, not so much, I imagine, to errors of observations as to the excessive amplitude of vibration, involving a departure from the frequency proper to infinitely small amplitudes."

Jet Break-up

When the aperture has the form of an exact circle, and when the flow of fluid in its neighbourhood is unimpeded by obstacles, there is a perfect balance of lateral motions and pressures, and consequently nothing to render the jet in its future course unsymmetrical. Even in this case, however, the phenomena are profoundly modified by the operation of the capillary force. Far from retaining the cylindrical form unimpaired, the jet rapidly resolves itself in a more or less regular manner into detached masses.

Eggers (1997) p 881

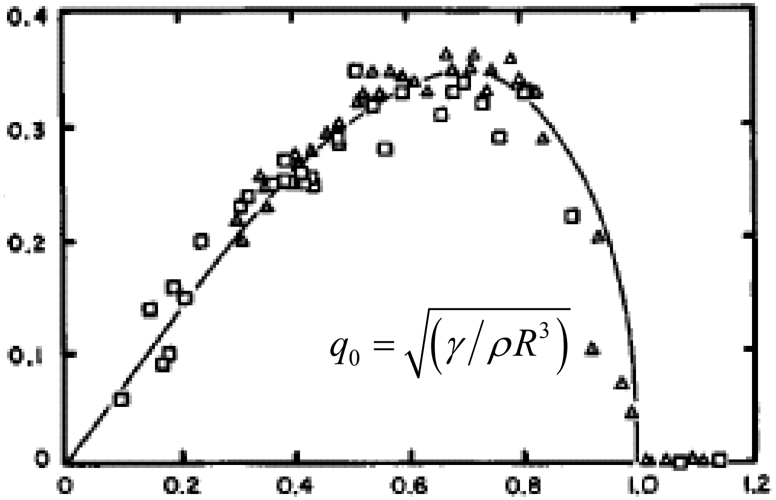
Data: Donnelly and Glaberson (1966); Goedde and Yuen (1970)

$$q = \left(\frac{T}{\rho a^3}\right)^{1/2} F(ka)$$

$$k = 2\pi\lambda^{-1} \quad (ka)^2 = 0.4858 \quad F(ka) = 0.343$$

$$\lambda_R = 9.0a$$

q/q_0



$x = 2\pi R/\lambda$

$q^{-1} \equiv$ Time taken for disturbance to get multiplied **1:*e***

For water $q^{-1} = 0.115 d^{3/2}$

For 1 cm diameter column of water the disturbance is multiplied 2.7 times in 0.115 seconds~1/9th of a second, and a 1000 fold increase happens in about 1/40th of a second.

Jet Break-up following Savart

- Savarts observations
  - Disturbances that upset the equilibrium is imposed on the fluid as it leaves the aperture.
  - Continuous portion of the jet is the distance traveled in time that is needed for the disintegration of the jet.
  - The continuous part can be extended by suitable shielding of the jet from external "tremors"

$$L_{jet} \propto \sqrt{head} \big/ diameter$$

(Imperical law: Savart)

If q, T, a known → k known → λ known

$$\lambda = \lambda_R$$

$$p = \frac{\sqrt{2hg}}{\lambda} \qquad p = \frac{\sqrt{2hg}}{4.5d}$$

Imposed vibrations

When vibrations are imposed at "*whose pitch is approximately proper to that of the jet*"

Imposed vibrations with a tuning fork and looked at shadow pictures using electric spark as flash.

The spark may be obtained from the secondary of an induction coil, whose terminals are in connexion with the coatings of a Leyden jar. By adjustment of the contact breaker the series of sparks may be made to fit more or less perfectly with the formation of the drops. A still greater improvement may be effected by using an electrically maintained fork, which performs the double office of controlling the resolution of the jet and of interrupting the primary current of the induction coil. In this form the experiment is one of remarkable beauty. The jet, illuminated only in one phase of transformation, appears almost perfectly steady, and may be examined at leisure.

- Jet issued horizontally and immediately assumed a rippled out-line
- Gradually increasing amplitude leads to the formation of elongated ligaments→ subsequently transformed into "spherules"
- "*In consequence of the transformation being at a more advanced stage at the forward than at the hinder end, the ligament remains for a moment connected with the mass behind, when it has freed itself from the mass in front, and thus the resulting spherule acquires a backward relative velocity, which of necessity leads to collision*."
- Drops rebound backwards and forward.

Imposed vibrations (contd)

$$q = \left(\frac{T}{\rho a^3}\right)^{1/2} F(ka)$$

$$q_{H_2O} = (9 \times 0.343)\, a^{-3/2}$$

$$ka < 1,\ \lambda > 2\pi a$$

Interfaces unstable: division occurs

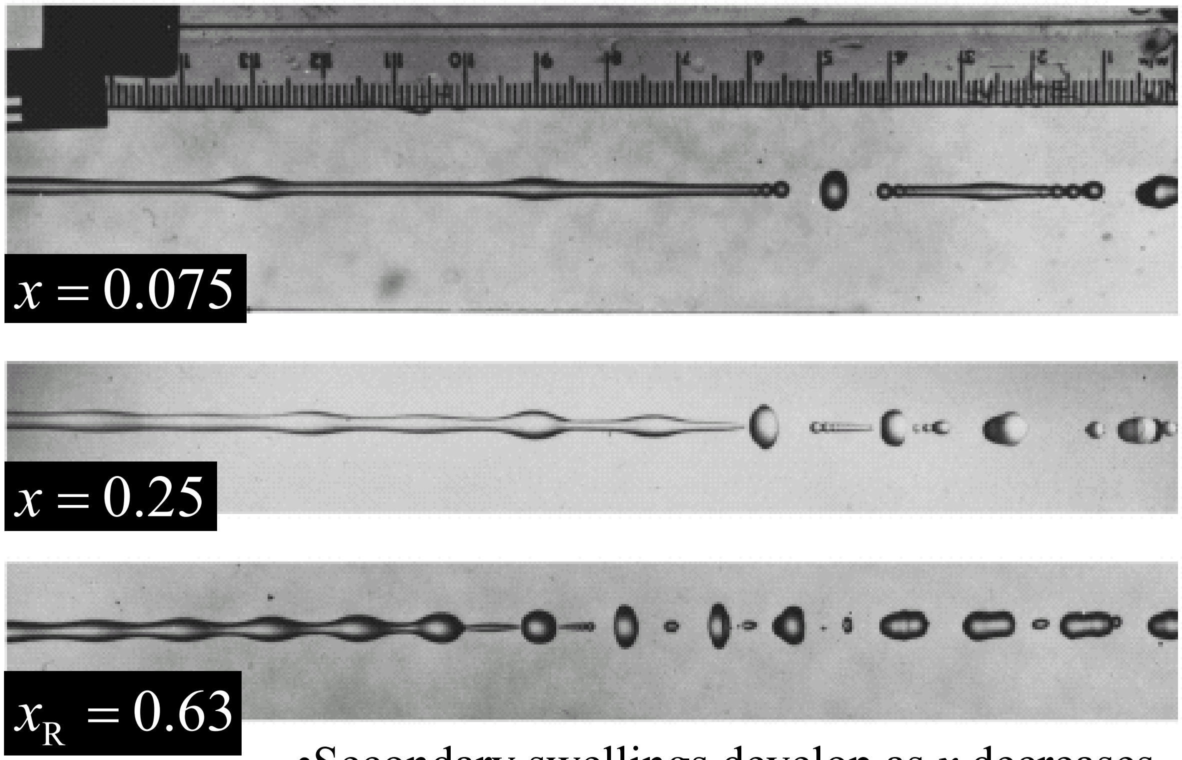
Interval defining upper limit

$$\lambda = \lambda_R = 4.5 \times 2a$$

$$\lambda/\lambda_R = \pi/4.5$$

$$(\pi : 4.5)$$

Rutland and Jameson (1971) $x = 2\pi R/\lambda$



- Secondary swellings develop as x decreases

Conclusions

- Examination of instability propagation on jets→ relates frequency and wavelength with jet area and pressure head.
- Dispersion relationship for jet break-up in the zero viscosity limit
- Observation of oscillations of the drops in phase with the frequency of break-up→ calculation of volume, surface area, and eccentricity of drop, drop oscillation time.
- Observation of satellite drop formation.